

A GAS OF D-INSTANTONS

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ABSTRACT

A D-instanton is a space-time event associated with world-sheet boundaries that contributes non-perturbative effects of order $e^{-const/\kappa}$ to closed-string amplitudes. Some properties of a gas of D-instantons are discussed in this paper.

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Properties of string theory are greatly affected by world-sheet boundary conditions. For example, a theory may be defined by summing over world-sheets with boundaries on which the string space-time coordinates are required to satisfy constant Dirichlet conditions – the entire boundary is mapped to a point in the target space-time and the position of that point is then integrated, which restores target-space translation invariance ([1] and references therein). The result is a theory that describes closed strings which possess dynamical point-like substructure as is indicated by the fact that fixed-angle scattering is power behaved as a function of energy. Recently an interesting variation of this scheme has been suggested [2] (based on [3]), involving the idea of ‘D-instantons’. As the name suggests, these are world-sheet configurations which correspond to a target space-time ‘event’, giving rise to exponentially suppressed contributions to scattering amplitudes behaving as $e^{-C/\kappa}$ (where κ is the closed-string coupling constant that is determined by the dilaton expectation value and C is a constant). This is in accord with general observations in [4] that suggest that whereas instanton effects in field theory typically behave as $e^{-const./\kappa^2}$, analogous effects in closed-string theory should behave as $e^{-C/\kappa}$. In this paper the single D-instanton contribution will be reexpressed as an exponential of an instanton ‘action’ and a gas of such instantons will be defined. The novel divergences associated with Dirichlet boundaries will be shown to cancel to all orders (as suggested in [2]). The leading contribution to the free energy comes from free D-instantons and is of order κ^{-1} but corrections due to long-distance interactions between D-instantons will be seen to be of order κ^0 .

A general oriented string world-sheet has an arbitrary number of boundaries and handles. In conventional open-string theories the embedding coordinates satisfy Neumann conditions on the boundaries while in the Dirichlet case each boundary is fixed at a space-time point, y_B^μ (where B labels the boundary), which is then integrated. The boundaries of moduli space are of various types. There are the usual degenerations of cylindrical segments that correspond to the propagation of physical closed-string states:

- (a) Degeneration of handles.
- (b) Degeneration of trivial homology cycles that divide a world-sheet into two disconnected pieces.

In addition, in the presence of boundaries there are degenerations of the following kinds:

- (c) A boundary may shrink to zero length giving the singularities associated with closed-string scalar states coupling to the vacuum through the boundary.
- (d) Degeneration of strips forming open-string loops. If both string endpoints are fixed at the same target-space point this gives an infinite contribution.
- (e) Degeneration of trivial open-string channels, in which the world-sheet divides into two disconnected pieces. In the case of Dirichlet conditions the intermediate open string necessarily has both end-points fixed at the same space-time point leading to another infinite contribution.

The cohomology of the states of the open-string sector where the string end-points are fixed at y_1^μ and y_2^μ is isomorphic to that of the usual Neumann open string with momentum $p^\mu = \Delta^\mu \equiv y_2^\mu - y_1^\mu$. This may be viewed as a simple consequence of target-space duality and it means that the arbitrary diagram possesses a rich spectrum of

space-time singularities, just as the usual loop diagrams possess a rich momentum-space singularity structure. However, it is important to realize that the wave functions of these states depend on the mean position, $y^\mu = (y_1^\mu + y_2^\mu)/2$, in addition to Δ^μ – this extra variable has no analogue for the usual open strings. The intermediate open string in the trivial degeneration (e) has $\Delta = 0$ so its cohomology is isomorphic to that of the usual Neumann open-string theory when $p^\mu = 0$. There is only one physical state in this case, which is the level-one vector. This is the isolated zero-momentum physical state with a constant wave function in the usual theory. However, in the Dirichlet theory its wave function $\zeta^\mu(y)$ is an arbitrary function that is physical without the need to impose any constraints on it – it is a target-space Lagrange multiplier field. The presence of this as an intermediate state in a string diagram leads to a divergence (the propagator for the level-one state is singular since a Lagrange multiplier field has no kinetic terms). The vertex operator that describes the coupling of this level-one vector state to a boundary is given by

$$g \oint d\sigma_B \zeta \cdot \partial_n X(\sigma_B, \tau_B) = ig \zeta^\mu \frac{\partial}{\partial y_B^\mu}, \quad (1)$$

where τ_B is the world-sheet position of the boundary and it is fixed at y_B in the target space (and g is the open-string coupling constant that is proportional to $\sqrt{\kappa}$).

At present there are two schemes for dealing with this level-one divergence. In one of these the Lagrange multiplier field is eliminated by integrating it, thereby imposing a constraint before the perturbation expansion of the theory is considered. Some consequences of the presence of this constraint were discussed in [5]. The other scheme [2] uses combinatorics for the sum over boundaries that is different from that of [1]. In this case the divergences due to the level-one open-string field should cancel between an infinite number of diagrams as indicated in [2]. The divergences will be shown to cancel in general in the formulation of the multi D-instanton gas to be presented below.

As a preliminary, we shall obtain an equation for the contribution of the level-one divergences to a particular sum over connected (orientable) world-sheets with Dirichlet boundaries. Consider a connected orientable world-sheet with p_i boundaries fixed at any one of a finite number of points y_i (where $i = 1, \dots, n$ and $p_i = 0, \dots, \infty$). The string free energy,

$$f_{p_1, p_2, \dots, p_n}(y_1, y_2, \dots, y_n), \quad (2)$$

is given by the usual multi-dimensional integral over the moduli space of the surface which has a total of $\sum_i p_i$ boundaries. We shall be interested in the sum over surfaces with all possible numbers of boundaries for a given value of n ,

$$S^{(n)} = \sum_{p_1, p_2, \dots, p_n=0}^{\infty} \frac{1}{p_1! p_2! \dots p_n!} f_{p_1, p_2, \dots, p_n}(y_1, y_2, \dots, y_n), \quad (3)$$

where the explicit combinatorial factor accounts for the symmetry under the interchange of identical boundaries – boundaries that are fixed at the same space-time point. The definition of f_{p_1, \dots, p_n} implicitly contains a sum over handles and the term with all $p_i = 0$

in (3) is just the usual closed-string free energy, $S^{(0)}$. Both types of open-string degenerations, (d) and (e), described earlier lead to divergences in (3) due to the intermediate level-one open-string states and in each case the coefficient of the divergent term is proportional to the product of level-one vertex operators attached to the boundary at either end of the degenerating strip. It is sufficient to consider single degenerations since the multiple degenerations are a subspace of these. The divergences of interest have the form $\int_\epsilon^1 dq/q = \ln \epsilon$ where ϵ is a world-sheet regulator. Degenerations of type (e) divide the world-sheet into two factors, so that if the degenerating boundary is fixed at y_1^μ the singular term has the form,

$$\sum_{p_1, p_2, \dots, p_n; q_1, q_2, \dots, q_n} \frac{f_{p_1+q_1+1, p_2+q_2, \dots, p_n+q_n}}{(p_1+q_1+1)!(p_2+q_2)! \dots (p_n+q_n)!} \sim \ln \epsilon \left(\frac{\partial}{\partial y_1^\mu} \sum_{p_1, p_2, \dots, p_n} \frac{f_{p_1, p_2, \dots, p_n}}{p_1! p_2! \dots p_n!} \right) \left(\frac{\partial}{\partial y_{1\mu}} \sum_{q_1, q_2, \dots, q_n} \frac{f_{q_1, q_2, \dots, q_n}}{q_1! q_2! \dots q_n!} \right), \quad (4)$$

where the derivatives arise from two level-one open-string vertex operators attached to two different boundaries. A divergence also arises when an internal open-string with both ends fixed at the same point degenerates (this is a degeneration of type (d)). In this case the coefficient of the divergence is proportional to two vertex operators of the level-one state attached to the same boundary giving,

$$\sum_{p_1, p_2, \dots, p_n; q_1, q_2, \dots, q_n} \frac{f_{p_1+q_1+1, p_2+q_2, \dots, p_n+q_n}}{(p_1+q_1+1)!(p_2+q_2)! \dots (p_n+q_n)!} \sim \ln \epsilon \frac{\partial^2}{\partial y_1^2} \sum_{p_1+q_1, p_2+q_2, \dots, p_n+q_n} \frac{f_{p_1+q_1, p_2+q_2, \dots, p_n+q_n}}{(p_1+q_1)!(p_2+q_2)! \dots (p_n+q_n)!}. \quad (5)$$

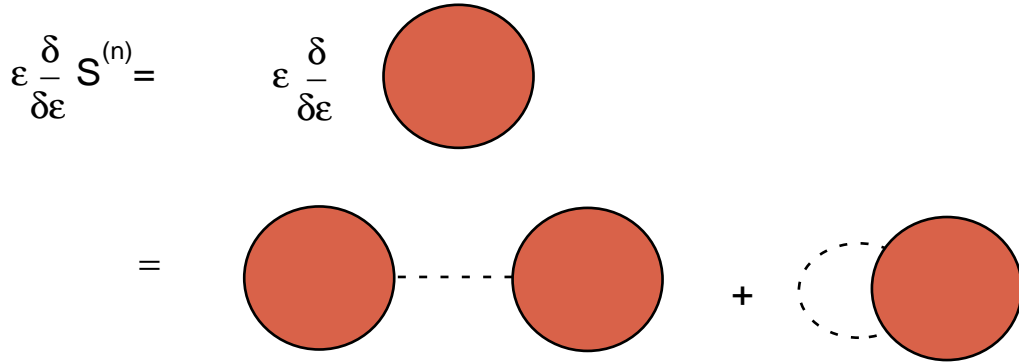


Fig. 1: The shaded blobs indicate a sum over world-sheets with boundaries inserted at any of the n positions, y_i ($i = 1, \dots, n$). The degeneration of a strip is indicated by a dashed line which either divides the surface into two (case (e)) or represents the degeneration of an internal open-string propagator (case (d)). This gives a divergence that is proportional to the product of the total momenta entering the boundaries at either end of the line.

Combining (4) and (5) and taking a derivative of the free energy with respect to ϵ extracts the dependence on the divergent degenerations as illustrated in diagrammatic form in fig. 1,

$$\epsilon \frac{\partial}{\partial \epsilon} S^{(n)} = \left(\frac{\partial}{\partial y_1^\mu} S^{(n)} \right) \left(\frac{\partial}{\partial y_{1\mu}} S^{(n)} \right) + \frac{\partial^2}{\partial y_1^2} S^{(n)}. \quad (6)$$

This equation is reminiscent of the renormalization group equation expressing the effect of closed-string divergences in [6].

The rules for constructing the string partition function in the presence of a single D-instanton may be abstracted from the rules given in [2] as follows. Firstly, sum over world-sheets with insertions of any number of handles and Dirichlet boundaries that are all at the *same point* in the target space, y_1^μ , which is to be integrated over. The sum is now taken to include *disconnected* world-sheets although these do not appear to be disconnected from the point of view of the target space since the boundaries all meet at the same point. A suitable symmetry factor is to be included to take account of symmetry under the interchange of identical disconnected world-sheets. The resulting one D-instanton partition function be expressed in exponential form as

$$Z^{(1)} = \int d^D y e^{S^{(1)}(y)}, \quad (7)$$

where $S^{(1)}(y) = S^{(0)} + \sum_{p=1}^{\infty} f_p(y)/p!$ is now interpreted as the one D-instanton ‘action’ that is given by the $n = 1$ term in (3). Scattering amplitudes may be generated from this expression if $S^{(1)}$ is taken to be a functional of the background fields.

The requirement of consistent clustering properties in the target space (as well as on the world-sheet) motivates the following generalization that includes the sum over an arbitrary number of D-instantons (and which should be equivalent to the rather schematic generalization motivated by duality in [2]). This involves summing over insertions of boundaries at any number of positions, y_i , that are to be integrated. The partition function is given in the language of a conventional instanton gas by the expression

$$Z = \sum_n \frac{1}{n!} \left(\prod_{i=1}^n d^D y_i^\mu \right) e^{S^{(n)}(y_1, \dots, y_n)}, \quad (8)$$

where $S^{(n)}$ is given by (3) and is now interpreted as the action for n interacting D-instantons. Recall that $S^{(n)}$ is defined by a functional integral over connected world-sheets and includes a term with no boundaries which is equal to $S^{(0)}$, the usual closed-string free energy. The expression for Z therefore has the form,

$$Z = e^{S^{(0)}} \sum_n \frac{1}{n!} \left(\prod_{i=1}^n d^D y_i^\mu \right) e^{S^{(n)'}(y_1, \dots, y_n)}, \quad (9)$$

where $S^{(n)'}$ is defined to be $S^{(n)}$ with the zero-boundary term missing. In the general term in the sum any boundary may be located at any one of the n target-space positions, y_i^μ ,

which are analogous to the collective coordinates describing the positions of instantons in quantum field theory. It is convenient to decompose $S^{(n) \prime}$ into those terms in which all boundaries are fixed at the same point (the free D-instanton terms), those at which the boundaries are fixed at two different points (two-body D-instanton interaction terms), those involving three points (three-body D-instanton interactions), and so on,

$$S^{(n) \prime}(y_1, \dots, y_n) = \sum_{i=1}^n R_1(y_i) + \sum_{i \neq j}^n R_2(y_i, y_j) + \sum_{i \neq j \neq k}^n R_3(y_i, y_j, y_k) + \dots \quad (10)$$

It is important in writing this series to recall that the definition of $S^{(n) \prime}$ includes terms in which any subset of the p_i are zero – these are terms that also contribute to the definition of $S^{(m) \prime}$ with $m < n$.

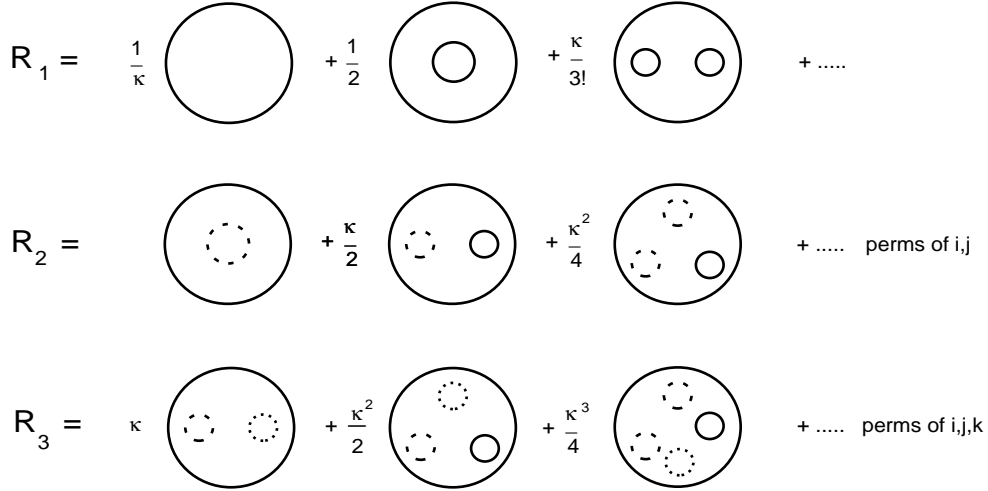


Fig. 2: Contributions to the n D-instanton action from the first few powers of κ . a) Diagrams with boundaries fixed at a single space-time point contribute to the free action. b) Diagrams with boundaries fixed at two points (indicated by the full and dashed boundaries) contribute to the two-instanton interaction. c) Diagrams contributing to the three-instanton interaction (at points indicated by full, dashed and dotted boundaries). The coefficients explicitly show the combinatorial factors that arise from symmetry under the interchange of identical boundaries.

The free term in (10), $R_1(y_i) \equiv S^{(1) \prime}(y_i)$, is simply given by the sum over connected orientable world-sheets of arbitrary topology with all boundaries fixed at a single point, y_i , illustrated in fig. 2(a). It is independent of y_i (by translational invariance) and has the form,

$$R_1 = -\frac{C}{\kappa} + \ln D + O(\kappa), \quad (11)$$

and thus $\sum_i R_1(y_i) = nR_1$. The term in this series with constant coefficient, C , is determined by functional integration over the disk. Explicit calculations determine $C =$

$2^8 \pi^{25/2} \alpha'^6$ [7,8]. It is somewhat remarkable that C is a finite (positive) constant with a value that is consistent with the non-vanishing value of the disk with a zero-momentum dilaton attached. Naively C would be expected to vanish since it should be proportional to the inverse of the volume of the conformal Killing group, $SL(2, R)$ (which is infinite) but that would not be consistent with the disk with a zero-momentum dilaton insertion. The ζ function regularization in [7] and the proper-distance regularization in [8] lead to the subtraction of an infinite constant from the volume of the Killing group, giving the finite positive renormalized value of C . The κ -independent constant D in (11) comes from the world-sheet annulus with both boundaries at y^μ .

Equation (11) leads to $e^{-C/\kappa}$ contributions to the partition function and to scattering amplitudes. This has the qualitative form expected for non-perturbative effects in string theory on the basis of matrix models and from the analysis of the rate of divergence of closed-string perturbation theory [4]. It is to be contrasted with a characteristic feature of non-perturbative effects (such as instantons and solitons) in field theory, which behave as $e^{-const./\kappa^2}$. This distinction between the non-perturbative behaviour of quantum field theory and that expected in closed-string theory seems likely to be of great significance (some possible consequences are described in [9]).

The two-instanton interactions are given by the series of terms in R_2 shown in fig. 2(b). The diagrams contributing to R_2 are those in which at least one boundary is fixed at either of the two space-time points. The leading terms in this series have the form

$$R_2(y_i, y_j) = f_{1,1}(y_i, y_j) + \frac{\kappa}{2} (f_{2,1}(y_i, y_j) + f_{1,2}(y_i, y_j)) + O(\kappa^2), \quad (12)$$

where $f_{p_i, p_j}(y_i, y_j)$ indicates a term with all $p_r = 0$ apart from p_i and p_j . The two-boundary term is given by the expression

$$f_{1,1}(y_i, y_j) = c \int_0^\infty d\tau e^{-\Delta_{ij}^2/\tau} e^{2\tau} \prod_{n=1}^\infty (1 - e^{-2\tau})^{-24}, \quad (13)$$

where c is a constant and $\Delta_{ij} = y_2 - y_1$. This diverges at the endpoint $\tau \rightarrow \infty$ due to the presence of a closed-string tachyon state. This is a familiar problem of the bosonic theory which we shall bypass by transforming to momentum space and declaring that at low momenta (or large distance) only the massless dilaton singularity survives so that in the long-distance limit $\Delta^2 \rightarrow \infty$

$$f_{1,1}(y_i, y_j) \sim |y_i - y_j|^{2-D}. \quad (14)$$

This coulomb-like behaviour due to dilaton exchange is analogous to the long-distance force between two classical instantons (magnetic monopoles) in the three-dimensional euclidean Georgi–Glashow model. However, unlike the case of magnetic monopoles the interaction term, (14) is not of the same order in κ as the leading term, $S^{(1)'}$.

It is straightforward to show that the partition function defined by (9) does not have the divergences arising from the level-one open-string vector state. The term in the partition function coming from n D-instantons has a dependence on ϵ that can be written by expanding the exponent to first order in $\ln \epsilon$ using (6), giving,

$$\ln \epsilon \int \prod_{i=1}^n d^D y_i^\mu \left\{ \left(\frac{\partial}{\partial y_1^\mu} S^{(n)} \right) \left(\frac{\partial}{\partial y_{1\mu}} S^{(n)} \right) + \frac{\partial^2}{\partial y_1^2} S^{(n)} \right\} e^{S^{(n)}} = 0. \quad (15)$$

The fact that the expression vanishes makes use of an integration by parts of the second term.

The cancellation is illustrated in an example in fig. 3. This shows contributions to a particular divergence coming from the sum of (a) a planar connected world-sheet, (b) a planar disconnected world-sheet and (c) a non-planar disconnected world-sheet. The sum of these contributions may be written symbolically as

$$(a) + (b) + (c) = \ln \epsilon \int d^D y_1^\mu \frac{\partial^2}{\partial y_1^2} (f_{1,1}(y_1, y_2) f_{2,1}(y_1, y_2)), \quad (16)$$

which vanishes after integration over y_1 (assuming suitable boundary conditions). The cancellation of divergences evidently involves a conspiracy between terms with different numbers of boundaries and handles. Therefore it is only possible when the boundary weight has a specific value – it is not possible to add Chan–Paton factors to the boundaries as is usually the case in open string theories. It is disturbing that the cancellation of the divergences requires an integration by parts which looks nonlocal since the D-instanton gas is supposed to satisfy clustering properties that express the locality of the theory. However, (at least in flat space) the potentially dangerous surface terms that arise are suppressed since they involve interactions between boundaries fixed at widely separated points.

On-shell scattering amplitudes may be defined in the usual manner by considering fluctuations in the background fields, resulting in closed-string vertex operators coupled to the world-sheets. The connected amplitudes are obtained from $\ln Z$, where the word ‘connected’ refers to the target space. It is straightforward to show, closely following the usual field theory combinatoric arguments, that $\ln Z$ generates only amplitudes in which all vertex operators are attached to world-sheet components with at least one boundary fixed at a common target-space point. These represent terms that are connected in the target space even though they involve disconnected world-sheets. The terms that are disconnected in the target space (terms in which vertex operators are attached to disconnected world-sheet components that do not have any boundary fixed at a common point) cancel out of the expansion of $\ln Z$.

As an example, we may ask how light particle masses depend on the presence of the boundaries. Thus, the mass of the dilaton is given by the dilaton two-point function evaluated at a minimum of the dilaton potential, assuming there is one. The leading contribution from a single D-instanton comes from the diagram in which the two dilaton

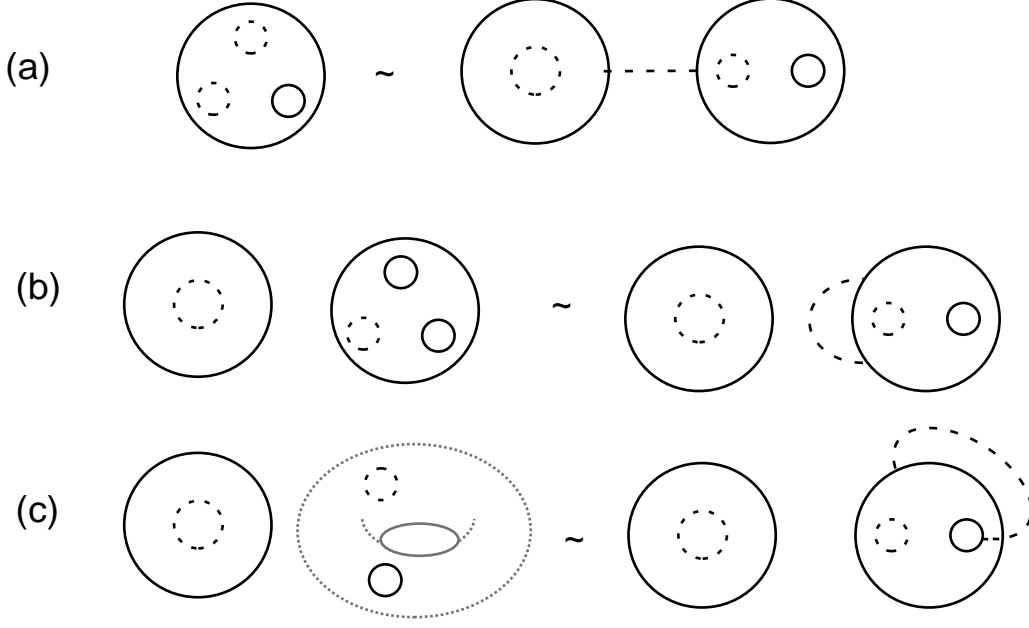


Fig. 3: An example of the cancelation of a divergence that requires world-sheets with handles. a) One of the divergent degenerations of a world-sheet with two boundaries at y_1 (full lines) and two at y_2 (dashed lines). b) A degeneration of a disconnected planar world-sheet that, after integration over y_1 , contributes to the same divergence as in a). c) A degeneration on a disconnected world-sheet with a handle that gives a divergence that adds to the divergences in b) and c) to give a total y_1 derivative.

vertex operators are inserted in disconnected disks with boundaries fixed at a common target-space point, y^μ , which is integrated. The result is of the form $(\kappa)^0 e^{-R_1/\kappa}$ where R_1 is given by (11). However, at this level of understanding the dilaton potential does not have a minimum at a finite value of the dilaton field so that the value of κ is incorrect and this term has no relation to the actual dilaton mass.

Now consider a process with two on-shell closed-string second-rank massless tensor states with polarization tensors $\zeta_{\mu\nu}^{(i)}$ and momenta $p^{(i)\mu}$ ($i = 1, 2$) satisfying the covariant conditions,

$$p^{(i)\mu} \zeta_{\mu\nu}^{(i)} = p^{(i)\mu} \zeta_{\nu\mu}^{(i)} = 0, \quad \zeta^{(i)\mu}{}_\mu = 0, \quad p^{(i)2} = 0. \quad (17)$$

The lowest-order single D-instanton contribution with these external states comes from a diagram in which both vertex operators are attached to the same disk with the boundary fixed at y^μ . Before integration over y^μ this defines a form factor that can be expressed as $\kappa D e^{-C/\kappa} e^{iq \cdot y} F(1, 2; q)$ (where $q^\mu = p^{(1)\mu} + p^{(2)\mu}$) with $F(1, 2; q)$ given by the remarkably simple gauge invariant expression [10],

$$F(1, 2; q) = \zeta^{(1)\mu\nu} \zeta_{\mu\nu}^{(2)} \frac{1}{q^2 - 8} + \zeta^{(1)\mu\nu} \zeta_{\nu\mu}^{(2)} \frac{1}{q^2 + 8} - \frac{1}{2} \zeta^{(1)\nu\mu} \zeta_{\rho\nu}^{(2)} q^\rho q_\mu \left[\frac{1}{q^2} - \frac{1}{q^2 + 8} \right] \\ + \frac{1}{2} \zeta^{(1)\mu\nu} \zeta_{\rho\nu}^{(2)} q^\rho q_\mu \left[\frac{1}{q^2} - \frac{1}{q^2 - 8} \right] + \frac{1}{16} \zeta_{\mu\nu}^{(1)} \zeta_{\rho\sigma}^{(2)} q^\rho q^\sigma q^\mu q^\nu \left[\frac{1}{q^2 - 8} - \frac{2}{q^2} + \frac{1}{q^2 + 8} \right]. \quad (18)$$

With external gravitons $\zeta^{(i)\mu\nu}$ is symmetric and the result vanishes after the y^μ integration (which sets $q^\mu = 0$), so the graviton does not develop a mass. The terms with poles at $q^2 = 0$ arise from the coupling of the dilaton to the vacuum and would be absent if the vacuum state were defined properly as a minimum of the dilaton potential.

The lowest-order contribution to the scattering amplitude with four gravitons comes from the diagram in which a pair of vertex operators are coupled to one world-sheet disk and another pair to a different disk where the boundaries of the two disks are fixed at the same point, y^μ , that is integrated. The amplitude is therefore given by

$$A = \kappa^2 D e^{-C/\kappa} \int d^D y F(1, 2; y) F(3, 4; y), \quad (19)$$

which is proportional to the convolution of two form factors.

The inconsistencies of the critical bosonic string make it difficult to interpret this theory in more detail. In particular, it is not at all clear in what sense these ideas make contact with more conventional instanton ideas, such as those that arise in matrix models. It would be of interest to study similar boundary effects in two-dimensional bosonic string theories and compare them with other descriptions of instantons in such theories (such as [11]). One peculiarity, at least in the bosonic theory, is that there are no anti D-instantons. The absence of the tachyonic closed-string singularity in the superstring also suggests that this may be an arena where a more consistent discussion could be given [12]. The disk diagram within superstring theory is again a finite constant, given by the non-zero coupling of the dilaton to the disk with one particular spin structure of the world-sheet fermions.

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